Elucidation of the idea of some strong NP-hardness proofs of scheduling problems

Radosław Rudek

Wrocław University of Economics, Poland, e-mail: radoslaw.rudek@ue.wroc.pl

1 Introduction

We have been pointed out that by omitting in our papers [1]-[5] some original description in the strong NP-hardness proofs, we may confuse readers and suggest them an existing of some logic errors. Our mistake was that we omitted in the parts "If" of the proofs in the related papers a description of assumptions of general cases (here denoted by (a') and (b')). Instead of that we left relevant calculations and cases (a) and (b). Although they have the smallest (as required) contribution to the criterion values in the provided instances for the strong NP-hardness proofs, they may be confusing.

Although we have presented one elucidation, we have been pointed out that it may be perceived as too large. Thus, we will present its minimized version. Namely, below we restore the omitted parts of the original proofs, which should make the proofs clear and also logically consistent. Note that there are no new calculations, only the presentation of the proof (idea and assumptions) is slightly extended.

2 The maximum lateness minimization [1], [2], [3], [5]

In the proof (part "If") [1], it is more appropriate to move cases (a) and (b) at the end of the proof. At the beginning instead of them, there should be the following more general (a') and (b'):

(a') $\lambda_u > 0$,

(b') $\lambda_u < 0$ and $\lambda_w > 0$ and $\sum_{l=u}^{i} \lambda_l < 0$ for $i = u, \ldots, w-1$ (and $\sum_{l=u}^{w} \lambda_l \geq 0$),

where $1 \leq u < w \leq m$ and for both $\lambda_i = 0$ for $i = 1, \ldots, u-1$. This should resolve any doubts.

However, for a better comprehension, we add the following description. In other words, (a') covers cases, where $\lambda_u > 0$ is the first (starting from 1 towards m) non-zero element; the values of λ_i for $i > u$ are arbitrary. On the other hand, (b') covers other cases. Namely, $\lambda_u < 0$ is the first (starting from 1 towards m) non-zero element and λ_w is the first element (starting from 1 towards m) such that $\sum_{l=u}^{w} \lambda_l \geq 0$. Thus, $\lambda_w > 0$ and $\sum_{l=u}^{i} \lambda_i < 0$ for $i = u, \ldots, w-1$; the values of λ_i for $i > w$ are arbitrary. It can be observed that (a') and (b') cover all possible distributions of λ_i for $i = 1, \ldots, m$.

In the calculation part of the proof for the original case (b), there should be $\sum_{i=u}^{w} \lambda_i \geq 0$ (page 500, line 14). However, the values of $C_{E_{w+1}}$ and C_{X_m} are further estimated from below for $\sum_{i=u}^{w} \lambda_i = 0$.

The original cases (a) and (b) should appear at the end of the proof as an additional observation and discussion (keeping in mind that the proof is done for (a') and (b')). These comments should include the observation that from the perspective of this proof, we have $L_{\max} > y$ regardless of λ_i for $i > u$ (case (a')) and for $i > w$ (case (b')). Moreover, for case (b'), the term $\sum_{i=u}^{w} \lambda_i = 0$ has the minimal contribution to $C_{E_{w+1}}$ and C_{X_m}.

Furthermore, the part (page 499, line -9) "*Consider case (a) and assume that (...). Thus, without loss of generality, we assume that* $\lambda_i = 0$ *for* $i \in \{1, \ldots, u-1\} \cup \{w+1, \ldots, m\}$" is not a proof that (a) and (b) cover all required λ_i. By this part, we wanted to say nothing more than that (a) and (b) can be analysed separately. Since it is quite obvious for (a) and (b) and it concerns only special cases (a) and (b), we admit that this part is redundant, thus confusing, and it may be removed.

The same elucidation concerns the proofs in [2], [3] and [5].

3 The makespan minimization [4], [5]

The cases (a) and (b) (part "If") in [4] are more appropriate to be moved at the end of the proof and instead of them, there should be more general (a') and (b'):

(a') $\lambda_w > 0$,

(b') $\lambda_u > 0$ and $\lambda_w < 0$ and $\sum_{l=u}^{i} \lambda_l > 0$ for $i = u, \ldots, w-1$ (and $\sum_{l=u}^{w} \lambda_l \geq 0$),

where $1 \leq u < w \leq m$ and for both $\lambda_i = 0$ for $i = w+1, \ldots, m$. This should resolve any doubts.

However, for a better comprehension, we add the following description. In other words, (a') covers cases, where $\lambda_w > 0$ is the first (starting from m towards 1) non-zero element; the values of λ_i for $i < w$ are arbitrary. On the other hand, (b') covers other cases. Namely, $\lambda_w < 0$ is the first (starting from m towards 1) non-zero element and λ_u is the first element (starting from m towards 1) such that $\sum_{l=u}^{w} \lambda_l \geq 0$. Thus, $\lambda_u > 0$ and $\sum_{l=u}^{i} \lambda_i > 0$ for $i = u, \ldots, w-1$; the values of λ_i for $i < u$ are arbitrary.

It can be observed that (a') and (b') cover all possible distribution of λ_i for $i = 1, \ldots, m$. Note also that they are symmetrical to the proof concerning the maximum lateness minimization.

In the calculation part of the proof for the original case (a), only the calculations for $C^{(2)}_{X_w}$ and $C_{\max}(\pi)$ are crucial (i.e., $C^{(2)}_{X_w} \geq S^{(1)}_{e_{w+1}} + V^{(2)}(X_w)$). Therefore, $C^{(2)}_{X_u}$ and $C^{(2)}_{e_{u+1}}$ should be removed; in fact, they are redundant even for original (a). Similarly for (b) in the relevant expressions the estimation "=" should be replaced with "$\geq$", e.g., $C^{(2)}_{X_u} \geq S^{(1)}_{e_{u+1}} + V^{(2)}(X_u)$.

The original cases (a) and (b) should appear at the end of the proof as an additional observation and discussion (keeping in mind that the proof is done for (a') and (b')). These comments should include the observation that from the perspective of this proof, we have $C_{\max} > y$ regardless of λ_i for $i < w$ (case (a')) and for $i < u$ (case (b')). Moreover, for case (b'), the term $\sum_{i=u}^{w} \lambda_i = 0$ has the minimal contribution to $C^{(2)}_{X_w}$ and $C_{\max}$.

Similarly as in the previous section, the part (page 26, line 1) "*Consider case (a) and assume that (...). Thus, without loss of generality, we assume that $\lambda_i = 0$ for $i \in \{1, \ldots, u-1\} \cup \{w+1, \ldots, m\}$.*" is not a proof that (a) and (b) cover all required λ_i. By this part, we wanted to say nothing more than that (a) and (b) can be analysed separately. Since it is quite obvious for (a) and (b) and it concerns only special cases (a) and (b), we admit that this part is redundant, thus confusing, and it may be removed.

The same elucidation concerns the proofs in [5].

4 Conclusions

The proofs (calculations) presented in [1]-[5] were done for the general cases (a') and (b') that cover all possible distributions of λ_i for "If". However, we have realized that some distributions of λ_i have the minimal contribution to the criterion value (i.e., (a) and (b)) and $L_{\max} > y$ or $C_{\max} > y$ regardless of the other values of λ_i. For instance if $L_{\max} > 0$ for (a) and (b), the same relation holds for (a') and (b'), respectively. Therefore, we have omitted (a') and (b'). Our intention was to have the same shape of the proofs such that they highlight some similarities (rules) between the proofs [1]-[3] and [4]-[5]. However, as we have been pointed out, it may confuse and suggest logical error. Thus, in this note, we restored the original shape of our proofs and additionally elucidated the original proofs.

Finally, one can observe that (b') can be further generalized to cover (a'), but the present form of the proofs seems to be more clear. Furthermore, the original cases (a) and (b) were more clear (and useful) to observe some dependencies (i.e., $\sum_{l=u}^{i} l\lambda_l$ in job completion times), which supported a construction (design) of a proper transformation; it was more comfortable for us to see some dependencies and to design the transformations.

References

[1] Rudek, R. (2012). Scheduling problems with position dependent job processing times: computational complexity results. Annals of Operations Research, 196, 491-516.

[2] Rudek, R. (2012). Some single-machine scheduling problems with the extended sumof- processing-time-based aging effect. International Journal of Advanced Manufacturing Technology, 59, 299-309.

[3] Rudek, R. (2012). The strong NP-hardness of the maximum lateness minimization scheduling problem with the processing-time based aging effect. Applied Mathematics and Computations, 218, 6498-6510.

[4] Rudek, R. (2011). Computational complexity and solution algorithms for flowshop scheduling problems with the learning effect. Computers & Industrial Engineering, 61, 20-31.

[5] Rudek, R. (2013). On single processor scheduling problems with learning dependent on the number of processed jobs. Applied Mathematical Modelling, 37, 1523-1536.