A critical note on "On a single machine-scheduling problem with separated position and resource effects"

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Abstract

This critical note concerns the paper [A. Janiak, M. Y. Kovalyov, M. Lichtenstein, On a single machine-scheduling problem with separated position and resource effects. Optimization, 2013, DOI:10.1080/02331934.2013.804077], where Janiak et al. provided selective and incorrect analysis of our clear and obvious proof of an optimality for a resource allocation algorithm. In the meaning of these authors, their analysis should show incorrectness of our proof. Furthermore, to make their claims stronger, they also provided a counter-example, which allegedly should prove the incorrectness of our resource allocation algorithm. Next, they presented the pseudocode, which in their meaning referred to a correct method. Nevertheless, we show that in fact, Janiak et al. omitted in their analysis significant and integral part of our proof, which is the source of their misleading and incorrect claims. Moreover, they analysed an algorithm that has nothing in common with the method presented in our paper, whereas our algorithm provides the correct result for their counter-example. Finally, the method published by Janiak et al. is exactly the pseudocode of the algorithm already described in our discussed paper, but they did not even mention about it.

Key words: machine scheduling; single machine; aging effect; resource allocation

1. Incorrect claims

The main topic of the paper by Janiak et al. [2] is to show that Rudek and Rudek [1] provided an incorrect proof of an optimal resource allocation property, i.e., the proof of Property 1 in [1]. In fact, Janiak et al. [2] selectively analysed only one part of our proof, from unknown reasons, omitting its integral part, which caused incorrectness of their claims, i.e.,

- Janiak et al. provided selective and incorrect analysis of our clear and obvious proof of optimality for a resource allocation algorithm (the proof of Property 1 in [1]). To support their point of view, they even added their own sentences as ours, which did not appear in our original proof. In fact, they omitted in their analysis significant and integral part of our proof, which is the source of their misleading and incorrect claims.
- Janiak et al. to make their claims stronger, they also provided a counter-example, which allegedly should prove the incorrectness of our resource allocation algorithm. In fact, they analysed an algorithm that has nothing in common with the method presented in our paper, whereas our algorithm (given in Property 1) provides the correct result for their counter-example.
- Janiak et al. presented the pseudocode of the resource allocation algorithm, which in their meaning is correct. In fact, it was exactly a pseudocode of the algorithm already described in our Property 1, but Janiak et al. did not even mention about it.

Although the discussed proof of an optimal resource allocation property (Property 1) seems to be simple and obvious, but taking into consideration the authors [2] do not fully follow it or read only its part, we explain the proof very carefully.

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2. Selective and incorrect analysis

Let us recall the property and some parts of its proof (see [1]), that are claimed by Janiak et al. to be incorrect.

Property 1 *The problem* $1|p_j(W(v), u_j) = p_j(W(v)) - a'_j u_j, \sum u_j \leq \hat{R}|C_{\max}$ *can be solved optimally by finding a permutation* π *that minimizes* $\sum_{i=1}^{n} p_{\pi(i)}(W(i))$*, and then by allocating the resource to the maximal possible amount till resource depletion, according to the non-increasing values of the resource ratio* a'_j*.*

The proof of this property (resource allocation) consists of two integral parts: (i) at first, we prove that the resource is allocated to jobs **according to the non-increasing order of** a'_j **parameter**; (ii) on the basis of (i), we further prove that the resource is allocated **up to the maximum possible amount until resource depletion** and the allocation is done according to the non-increasing order of a'_j.

Note that in part (i), for $a'_h < a'_k$ and $u_h > u_k$, we analyse cases: (a) u_h and u_k are allocated to jobs h and k, respectively (i.e., $a'_h u_h$ and $a'_k u_k$); (b) u_h and u_k are allocated to jobs k and h (interchanging), respectively (i.e., $a'_h u_k$ and $a'_k u_h$). Obviously, in both cases the allocated amounts of the resource u_h and u_k are not greater than the maximum possible amount of the resource that can be allocated to jobs h and k (i.e., we allocated u_h as well as u_k, see calculations in [1]). Following it, we prove that in the optimal solution if $a'_h < a'_k$ and $u_h > u_k$, then u_h is allocated to k and u_k is allocated to h. However, Janiak et al. [2] falsify put claims that have not appeared in our proof, i.e., Janiak et al. said that we:

"*try to show that interchanging the resource amounts of the jobs h and k decreases the* $C_{\max}$ *value, and come to a conclusion that the original resource allocation is not optimal and that* $a_h < a_k$ ***should imply*** $u_h \geq u_k$ ***in any optimal resource allocation****.*"

Note that it is completely different than the conclusion in our proof. In part (i), we do not claim that "$a_h < a_k$ **should imply** $u_h \geq u_k$ in **any optimal resource allocation**"! We only show that for $a_h < a_k$ the following resource allocation $u_h > u_k$ cannot be optimal for cases analysed in part (i). But this part do not take into consideration cases, where u_h or u_k is greater than the maximum possible amount. Such cases are analysed in (ii), where we take into considerations only solutions consistent with (i). But part (ii), which is integral element of our proof, for unknown reasons was completely omitted by Janiak et al. [2]. It can be easily observed that in the optimal solution the resource is allocated to the maximal possible amount till resource depletion, according to the non-increasing values of the resource ratio a'_j, which follows from two parts (i) and (ii).

3. Incorrect counter-example

Let us analyse the counter-example given by Janiak et al. [2] to show incorrectness of their claims.

"*There are* $n = 2$ *jobs with the following parameters:* $a_1 = 1$*,* $a_2 = 2$*,* $\bar{u}_1 = 3$*,* $\bar{u}_2 = 1$*, and* $R = 3$*. The only optimal resource allocation is* $u^* = (2, 1)$*, which contradicts authors' [Rudek and Rudek]* ***statement that*** $a_1 < a_2$ ***should imply*** $u_1^* \leq u_2^*$*.*" [2]

At first note that **we do not claim that** $a_1 < a_2$ **should imply** $u_1^* \leq u_2^*$! Let us apply our Property 1 to this "counter-example". Although Property 1 seems to be obvious, knowing that some of the authors do not fully follow it, we explain this property very carefully. Namely, we allocate the resource in the maximum possible amount according to the non-increasing order of a'_j. Since $a_1 < a_2$, then we allocate the resource first to job $j = 2$ and next to job $j = 1$. Property 1 says that the resource is allocated in the maximum possible amount, therefore, $u_2^* = 1$, since $\bar{u}_2 = 1$ and $R = 3$ is sufficient. Next we allocate resource to job $j = 1$ (since $a_1 < a_2$) in the maximum possible amount till resource depletion. Note that $\bar{u}_1 = 3$, but only $R = 2$ left. Thus, we allocate $u_1^* = 2$ since the resource is depleted. Following Property 1 the optimal resource allocation to jobs is $u^* = (u_1^*, u_2^*) = (2, 1)$, which is the same as the optimal solution provided by Janiak et al. However, we have no idea, which property Janiak et al. analysed and showed to be incorrect. We know that it has nothing in common with Property 1 published in our paper [1], which provides the optimal solution.

4. Repeating known results

Janiak et al. [2] claimed that they showed incorrectness of our Property 1, which was the reason that they provided optimal resource allocation algorithm called "Greedy algorithm". However, this algorithm is well known in the literature to be optimal for some resource allocation problems. Namely, the algorithm provided by Janiak et al. is nothing more than the pseudocode of the algorithm described in Property 1. Nevertheless, Janiak et al. did not even mention about it in [2]. Let us analyse the pseudocode given by Janiak et al. (see Algorithm 1).

Algorithm 1 Greedy algorithm

Step 1. Re-number the jobs such that $a_1 \geq \ldots \geq a_n$. *Set* $u_j = 0$, $j = 1, , \ldots, n$. *Set* $k = 1$.

Step 2. Reset $u_k := \min\{\bar{u}_k, R\}$ *and* $R := R - u_k$. *If* $R = 0$, *then stop. Otherwise, reset* $k := k + 1$, *and repeat Step 2.*

Obviously in Step 1 **jobs** are renumber such that they **are processed in the non-increasing order of** a'_j. In Step 2 the **resource is allocated according to the maximum possible amount** (i.e., $u_k := \min\{\bar{u}_k, R\}$) **till the resource depletion** (i.e., $R := R - u_k$. If $R = 0$, then stop). Note this is exactly, what Property 1 says. It is so obvious how this algorithm can be coded, thereby we did not artificially increase the size of our paper, and gave it only as the text in Property 1. We have no idea that there can be a problem to follow a clear procedure.

5. Conclusions

Janiak et al. [2] published the falsified claims, which are based on their own selective and incorrect analysis of clear and obvious results. Nevertheless, we showed that in fact, Janiak et al. omitted in their analysis the significant and integral part of our proof in [1], which is the source of their misleading and incorrect claims. Moreover, they analysed an algorithm that had nothing in common with the method presented in our paper, but we showed that our algorithm provides the correct result for their counter-example. Finally, we revealed that the correct resource allocation algorithm published by Janiak et al. was a pseudocode of the algorithm already described in our paper [1], but they did not even mention about it.

References

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